B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Physics

(Classical Dynamics)

Paper : DSE-2(3) (OR)

Full Marks: 60

Time: 3 Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group-A

2×10=20

1. Answer *any ten* questions from the following:

(a) Prove that a charged particle moves in a uniform magnetic field at constant magnitude of velocity.

(b) Obtain the expression of the generalized force. What can you say about its dimension?

(c) The Lagrangian of a system is $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - \mu r\sin\theta\sin\phi$. Find the conserved quantities.

(d) If the Lagrangian of a system does not depend upon time explicitly show that the Hamiltonian of the system is a constant of motion.

(e) The Hamiltonian $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$ where a and b are constants. Show that q_1q_2= constant.

(f) What are the normal frequencies and the normal coordinates of a coupled vibration?

(g) Write down the secular equation for a Lagrangian of three degrees of freedom,

$$L = \frac{1}{2}(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2) - \alpha^2(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3).$$

(h) A rod of length 1m is moving in the direction of its length, with velocity 0.8c along x-axis. Find the length as measured by an observer at rest in the lab frame.

(i) At what speed the total energy of a particle is n times its rest energy.

(j) Show that the space-time interval in four-dimensional space is an invariant quantity under Lorentz transformation.

(k) Obtain the equation for unit length in S' frame which moves with velocity v along x-axis of S frame in Minkowski space.

(l) What is space-like interval? Give its physical interpretation.

(m) Prove that four-vector force and four-vector velocity are orthogonal to each other.

(n) What is Reynold's number? Explain its significance.

(o) A flat plate of area 0.02 m^2 is separated from a large flat surface by a film of oil of uniform thickness 1 mm and viscosity 2 N-s/m^2. Determine the force required to slide the plate over the surface at a velocity of 4.5 cm/s.

5×4=20

2. Answer *any four* questions from the following:

(a) A charge particle starts moving from origin with an initial velocity in a plane perpendicular to a uniform magnetic field. Show that the path of the particle is helix. Also find the frequency of the revolution of the particle. 4+1=5

(b) (i) Find the equation of motion of a particle under central force field using Hamiltonian method.

(ii) What is cyclic coordinate? Show that the conjugate momentum corresponding to a cyclic coordinate is constant of motion. 3+2=5

(c) The Lagrangian of a system is $L = \alpha\dot{q}^2 - \beta \cos q$ where α and β are positive constants. Find the values of q for stable and unstable equilibrium. Obtain the Lagrange's equation of motion for small oscillation near equilibrium.

(d) (i) Obtain the expression of four-vector force in terms of force and velocity vector.

(ii) Consider a particle with rest mass m_0 is moving along x-direction with a speed v. The total energy is then $E = \gamma m_0 c^2$. Show that the momentum in the x direction is $\gamma m_0 v$. 3+2=5

(e) Using the Lorentz transformation of four-momentum vector obtain the expression for relativistic Doppler effect. Calculate the Doppler shift in wavelength for light of wavelength 6000Å when the source approaches the observer at velocity 0.2 c. 3+2=5

(f) (i) Obtain an expression of velocity profile of a liquid in laminar flow through capillary tube.

(ii) If two capillary tubes of radii r_1, r_2 and lengths l_1, l_2 respectively are connected in series find an expression of rate of flow of liquid using Poiseuille's equation. 3+2=5

3. Answer *any two* questions from the following: 10×2=20

(a) (i) State Hamilton's principle. Hence, derive Euler-Lagrange's equation.

(ii) A projectile of mass m is thrown at initial velocity $\vec{u}_0$ at inclination α with the horizontal plane. Write down the Lagrangian of the projectile. Hence, obtain the Lagrange's equation of motion and the equation of the path. (2+4)+(1+1+2)=10

(b) (i) Prove that the Lorentz transformation is the transformation from orthogonal system to non-orthogonal system in Minkowski space. Explain the length contraction using geometric representation.

(ii) Write down the expression of proper time interval. Hence, obtain the expression of velocity four-vector. Show that norm of this vector is time-like. (3+3)+(1+2+1)=10

(c) (i) From the definition of four-momentum vector show that $E^2 = p^2c^2 + m_0^2c^4$.

(ii) Obtain the expressions of total energies of product particles in two-body decay process in terms of their rest masses. Hence, show that the total kinetic energy of the product particles is the energy due to difference in mass of decaying particle and that of the product particles.

(iii) The decay of pion is $\pi^+ \rightarrow \mu^+ + \nu_\mu$. Calculate the momentum of μ^+ from the given data $[m_\pi = 0.1396GeV,\ m_\mu = 0.1057GeV,\ m_\nu = 0]$. 3+(3+2)+2=10

(d) (i) Derive the equation of continuity for fluid in motion. In a two dimensional fluid motion the velocity components are $u = -\frac{ay}{x^2+y^2}$, $v = \frac{ax}{x^2+y^2}$, $w = 0$. Show that the flow of fluid is possible. Also find the equation of stream line.

(ii) Consider tri-atomic molecule CO_2 as a linear coupled harmonic oscillator where two equal masses at two ends are connected by two springs of equal spring constant with another mass. All masses vibrate in longitudinal direction. Obtain the normal frequencies. (3+3)+4=10

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